Morphological characterization of dithering masks

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Abstract. *We present some novel tools for the analysis of blue-noise binary patterns. Unlike most of the existing methods that evaluate the frequency content of a given mask or its lower order statistics, our new metrics characterize the morphological content of a mask that is quanti®ed using simple one-pass ®ltering. An analytical ®lter expression is given. As a result, one can balance the structural content of the maskÐdiagonal, vertical, and horizontal interconnections of the majority (or minority) pixelsÐat the same level. In addition, it is possible to improve the overall mask quality by pre-*

$$AMD = \frac{\sum_{i=1}^{k} D_{min}(i)}{k} \qquad k = g * L, \tag{3}$$

where D_{min} denotes the minimal distance to the nearest neighbor for the i'th minority pixel, and k stands for the number of minority pixels at the gray level g in the mask that generates L color levels.

When considering two binary patterns that are both candidates for the gray-level approximation, we should choose the one with the bigger AMD value and, consequently, the less grainy of the two patterns. The AMD describes the binary pattern in term of graininess, but it does not reveal the exact nature (morphological shape) or position of a grainy artifact.

Although generally successful, the FWMSE and AMD (as well as other existing metrics) fail to localize (and sometimes even to recognize) problems at the mid-tone levels (output levels between 0.25 and 0.75) where the AMD is smaller than 2. Our proposed algorithm not only localizes such problems but also allows the efficient location of the exact position and morphological shape of a pixel "clump" (see Fig. 1).

3 Morphology Information Retrieval by Means of Filtering

In order to extract the morphological information from a certain gray level g as a result of the filtering process, one should construct a filter that has a unique response for each pixel configuration. For simplicity, we show as an example a very small filter size (2×2). However, since the filter construction process is generic, larger filters of this type could be used as an optimal look-up table (LUT) in a blue-noise mask (BNM) construction.

3.1 *Filter Construction*

Consider the binary pattern b to be filtered by a rectangular $M \times N$ filter f. The result is described by:

$$res(x,y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} f(m,n) \cdot b(x+m, y+n). \tag{4}$$

In order to obtain unique responses for each pixel configuration, we propose use of $M \times N$ filter f:

$$f(m,n) = 2^{m \cdot N + n} \qquad m = 0,1,\ldots,M-1 \quad n = 0,1,\ldots,N-1. \tag{5}$$

Equation (4) can be rewritten now as:

$$res(x,y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} 2^{m \cdot N + n} \cdot b(x+m, y+n). \tag{6}$$

If we recorder $b(x+m, y+n)$ in raster scan order as $b_{(x+m)N+(y+n)}$, then Eq. (5) can be rewritten as:

$$res(x,y) = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} 2^{m \cdot N + n} \cdot b_{(x+m)\cdot N+y+m}$$

For example, if 0 represents a white pixel and 1 represents a black pixel, then an ``upper" black horizontal connection will result in a ®lter output value of three ~1 1 2 5 3!. Thus, the ®lter output from a binary pattern as shown in Fig. 3 will contain unique numbers corresponding to the morphology of pixels within a sliding 3 22 window.

4 Morphological Characterization Algorithm

The extraction of morphological features using the described generic ®lter ~see Eq. ~5!# can be schematically represented as a two-step algorithm: the ®rst step being ®ltering ~circular convolution or correlation! and the second step being result identi®cation ~Fig. 4!.

As a result of the ®ltering process, we have a matrix of the ``morphological content" of a ®ltered binary pattern. Each value uniquely represents the content of the appropriate sliding window centered at the same pixel position as in the original ~binary! pattern. The second step, result identi®cation, is now simple. These values are used as pointers on the LUT with prede®ned actions for each pixel con®guration. In mathematical morphology this approach is known as a hit-or-miss transform.[9]

For the purpose of calculating the LUT index ~pointer!, the ®ltering step from Fig. 4 may be replaced by direct use of the mask binary values. The $n 3 n$ neighborhood of processed pixel may be reordered ~vectorized! as binary value $b_{n^2 2 1} b_{n^2 2 2} \ldots b2$

tial distribution, which can be calculated as spatial distribution of appropriate ®ltered values.

5 Metric Analysis

When considering a dithering mask at any given gray level g, it can be seen (Fig. 2) that there are a few basic groups of local pixel con®gurations: zero (all white), one-pixel, two-pixel horizontal, and two-pixel vertical and diagonal connections. The L-shaped connection is actually a onetypical BNM @Fig. 6~d!#, it is apparent that all of the WNM distributions are intersecting at one point ~output level 128!. That means there are equal numbers of all types of 2×2 con®gurations present at output level g=0.5. This results in visually disturbing pixel clumps ~all black! and spatial voids ~all white!. In the case of the BNM, the mask building algorithm tends to arrange minority pixels in certain patterns, resulting in the virtual nonexistence of all black and all white 2×2 con®gurations at the middle of the color scale. Also, the number of L-shaped connections is signi®cantly smaller than the number of any two-connections at the mid-tone color levels.

From Fig. 7, it is easy to locate the nature of nonoptimality of an analyzed BNM by inspection. In the mask building process, the original algorithm did not recognize that the number of horizontal and vertical rods ~values 3, 12, 5, and 10 in the morphological content matrix! became almost the same as the number of diagonal connections ~6 and 9! at an output level of 71. That characteristic propagated in the mask building process toward the lower part of the gray scale ~gray levels: g, 71/256). As a consequence, this particular dithering mask is better at the lighter mid-tones ~output levels in the range of 180 to 220 on this scale! than at the darker mid-tones ~output levels from 35 to 70!. The particular mask building algorithm used in this case failed to produce a completely balanced scale, thus affecting the overall dithering mask quality.

An example of this unbalance is given in Fig. 8, where two symmetrical gray levels ~205 and 50! are compared. The lighter one appears better, due to better balanced morphological content. The observed portion of the level 205 @Fig. 7~a!# has 6 vertical, 5 horizontal, and 15 diagonal connections versus 13 vertical, 15 horizontal, and 14 diagonal connections at the same-sized portion of the level 50 @Fig. 7~b!#. These numbers are consistent with the mask statistic shown in Figs. 7~a! and 7~b!.

If the information about the unbalance between symmetrical levels and unbalance in number of vertical, horizontal, and diagonal connections were used, the mask building algorithm would produce a better balanced mask.
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form!. One possible use of this ®lter is suggested. Its ability to identify the irregularities is demonstrated.

The analysis described in this work allows for the exploitation of certain morphological properties, characteristic in binary patterns, in order to evaluate the quality of mid-tone gray levels. Although the concept presented has been used as an analysis tool, it can be used for the morphological characterization and validation of a halftone mask as well as for the control part of the mask synthesis process.